Direct nuclear photoeffect in heavy deformed nuclei

B. S. Ishkhanov,[1,2] V. N. Orlin,[2] and K. A. Stopani[2]

[1]*Lomonosov Moscow State University, Department of Physics, Moscow, 119991 Russia*

[2]*Lomonosov Moscow State University, Skobeltsyn Institute of Nuclear Physics, Moscow, 119991 Russia*

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A model for direct photonuclear reactions on heavy deformed nuclei is formulated. This model is used for calculations of direct (γ, p) reactions on ^{160}Gd and ^{184,186}W. Importance of direct photoeffect for these nuclei at $E_\gamma \sim 30$ MeV is demonstrated.

INTRODUCTION

Photonuclear reactions induced by photons with energies $E_\gamma \lesssim 40$ MeV proceed mainly through formation of a compound nucleus. However, as measurements show, this photoabsorption mechanism only describes a part of the yield of photoprotons at the tail of the giant dipole resonance (GDR) from $A > 100$ nuclei. It indicates that at photon energies exceeding the GDR energy a direct photoeffect contribution to the (γ,p) reaction is very noticeable. In this case the photon energy is spent not on an excitation of a compound system, but on knock-out of a nucleon from the nuclear surface, and the resulting nucleus remains in the ground or a low-lying excited state.

When a nucleon is knocked out of the peripheral region of a nucleus following absorption of a photon with the orbital momentum $l \approx k_\gamma R$, where $k_\gamma = E_\gamma/\hbar c$ fm^{-1} is the transferred value of the wave vector, and $R = 1.2A^{1/3}$ fm is the nuclear radius. It is natural to expect that when l is approximately integer quasi-resonances appear in the direct photoeffect reaction cross section. The lowest lying "resonance" is expected at $l = 1$, which corresponds to electric dipole absorption and is placed at the energy of about $E_\gamma \sim 165\, A^{-1/3}$ MeV. Thus, a noticeable direct photonucleon yield in the $E_\gamma \sim 30$ MeV region should be expected for heavy nuclei with $A \sim 150$—200.

In the case of a spherical nucleus the direct photoeffect amplitude corresponding to knockout of a nucleon from a single-particle level jm can be approximated by a product $\langle f|a^+(jm)|i\rangle\langle \mathbf{k}^{(-)}|H_{\rm ptb}|jm\rangle_{\rm sp}$, where the first term is the matrix element of the $a^+(jm)$ operator for a transition from the $|i\rangle$ state of the initial nucleus to the $|f\rangle$ state of the final nucleus (genealogic coefficient), and the second term describes a single-particle nucleon transmission induced by electromagnetic field from the jm orbit to a stationary scattered state $|\mathbf{k}^{(-)}\rangle$ in the mean field of the final nucleus with waves of outgoing nucleon with a wave vector $\mathbf{k}$ converging at infinity. In order to calculate the genealogical coefficient $\langle|a^{+}(jm)|i\rangle$ one has to know the detailed structure of the states $|i\rangle$ and $|f\rangle$ which presents significant difficulties during direct photoeffect calculations in spherical nuclei.

This problem is non-existent for description of direct photoeffect in heavy deformed nuclei since the ground state wave function of the target nucleus in intrinsic coordinate system can be approximated as an anti-symmetrized product of single-particle nucleon states positioned at filled orbits of a deformed nuclear potential. However, due to deformation the step of calculation of scattered states of the outgoing nucleon in the mean nuclear field of the final nucleus becomes rather complicated.

Present work presents a detailed description of construction of scattered states in deformed mean field and an approximate model of direct photoeffect in heavy deformed nuclei due to electric dipole absorption is formulated.

I. ADIABATIC APPROXIMATION

We formulate the main assumptions that are used in description of the direct nuclear photoeffect:

1. only heavy axially symmetric deformed nuclei are considered;
2. the speed of rotational motion of the deformed nucleus and, thus, that of the potential is small in comparison with the speed of the outgoing nucleon knocket out of the nucleus by the photon, so it is not disturbed by this rotation (adiabatic condition);
3. it is assumed that the photon interacts only with one of the nucleons on outermost orbit of the valence shell. Other nucleons remain in their initial states, and the final nucleus is produced in ground state;
4. interaction of the electromagnetic field with the nucleus which results in direct photoeffect is considered in terms of time-dependent perturbation theory under the assumption that the main contribution to the reaction is from electric dipole absorption;
5. deformed optical potential is taken as the mean field in which the outgoing nucleon motion takes place (its construction is described below).

Within the adopted assumtions the differential cross section of direct photoeffect in the x', y', z' intrinsic frame, the z' axis of which is directed along the symmetry axis of the nucleus, can be represented as

$$\frac{d\sigma_{\rm int}(E_\gamma,\theta',\phi')}{d\Omega'} = \frac{4}{3}\pi^3\,\frac{mkE_\gamma e_{\rm eff}^2}{c\hbar^3}\times$$
$$\times\,\frac{1}{2}\sum_{\mu=\pm1}\sum_{s=\pm\frac{1}{2}}\left|\sum_\nu D^1_{\mu\nu}(\omega)\langle\!\langle(\mathbf{k}s)^{(-)}|r'Y_{1\nu}(\theta',\varphi')|\beta\rangle\right|^2, \tag{1}$$

where m is the reduced mass of the nucleon, $e_{\rm eff}$ is the effective nucleon charge (equal to eN/A for a proton, and $-eZ/A$ for a neutron), $|(\mathbf{k}s)^{(-)}\rangle\!\rangle$ is the scattering state in the mean field of a finite nucleus with vanishing at infinity waves, that corresponds to emission of a nucleon with the wave vector $\mathbf{k}$ and z' projection of spin of $s=\pm\frac{1}{2}$ (kak variant: s_z), $k=|\mathbf{k}|=\sqrt{2m\varepsilon}/\hbar$ is the absolute value of the wave vector of the outgoing nucleon, $\varepsilon=E_\gamma-B_{\rm thr}$ is its energy ($B_{\rm thr}$ is the nucleon separation energy in the target nucleus), ϑ',ϕ' are the polar and asymuthal nucleon emission angles relative to the intrinsic coordinate frame, $|\beta\rangle$ is the single particle state from which the nucleon is knocked out (since the target nucleus is axially symmetric it has a defined value m_β of angular momentum projection to the symmetry axis z'), $D^1_{\mu\nu}(\omega)$ is the finite rotation matrix describing conversion of the dipole moment operator from the intrinsic x',y',z' to the laboratory x,y,z frame, where the z axis corresponds to the direction of the incident photon, $\omega\equiv\theta_1,\theta_2,\theta_3$ are the Euler angles that describe the rotation of the intrinsic frame relative to the laboratory frame x,y,z, and the $\mu=\pm\frac{1}{2}$ quantum number account for two possible circular polarization states of the photon.

Summation in (1) takes into account different orientations of the spin of the outgoing nucleon and averaging over possible photon polarization states. — eto nado?

If the proton (neutron) number is even then the outer-most orbit contains two nucleons with $j_{z'}=\pm m_\beta$ each being knocked out by the photon with equal probability. This should imply two-fold increase of the cross section (1). However, due to pairing of the nucleons the probability that the nucleon pair takes the state ν close to the Fermi surface is less than 1. In deformed nuclei this probability can be approximated by the following expression [1]:

$$v_\nu^2 = \frac{1}{2}\left[1-\frac{\varepsilon_\nu-\lambda}{\sqrt{(\varepsilon_\nu-\lambda)^2-\Delta_\nu^2}}\right], \tag{2}$$

where ε_ν is the energy of the single-particle state, λ is the chemical potential, and Δ_ν is the pairing energy. Since the energy of the outermost orbit β is approximately equal to λ an approximate relationship $v_\beta^2\approx\frac{1}{2}$ holds for this state and no additional coefficient in eqref1 is required.

The $|(\mathbf{k}s)^{(-)}\rangle\rangle$ state can be expressed as a spherical harmonics expansion:

$$|(\mathbf{k}s)^{(-)}\rangle\rangle = \frac{\hbar}{\sqrt{mk}} \sum_{l=0}^{\infty} \sum_{j=|l-\frac{1}{2}|}^{l+\frac{1}{2}} \sum_{m=-j}^{j} (lm-s\,\frac{1}{2}s|jm) Y^{*}_{lm-s}(\theta',\phi')|\alpha^{(-)}\rangle\rangle, \tag{3}$$

where $|\alpha^{(-)}\rangle\rangle \equiv |(\varepsilon\, ljm)^{(-)}\rangle\rangle$ denotes scattering states in the nuclear mean field with the energy $\varepsilon = \frac{\hbar^2 k^2}{2m}$, which while not being eigenstates of $\mathbf{l}^2$ and $\mathbf{j}^2$ operators in the case of deformed field (and do not have corresponding definite quantum numbers l and j) have a special property that a wave packet built with them $\int\limits_{-\infty}^{\infty} \frac{\gamma/\pi}{(\varepsilon-\varepsilon_0)^2+\gamma^2} e^{-i\varepsilon t}|\alpha^{(-)}\rangle\rangle\, d\varepsilon$ at $t \to +\infty$ becomes a wave packet $\int\limits_{-\infty}^{\infty} \frac{\gamma/\pi}{(\varepsilon-\varepsilon_0)^2+\gamma^2} e^{-i\varepsilon t}|\varepsilon ljm\rangle\rangle_{\text{free}}\, d\varepsilon$ of freely moving nucleons with fixed l, j. With spheroidal deformation the $|\alpha^{(-)}\rangle\rangle$ states have also a definite parity π and projection m of the moment $\mathbf{j}$ onto the nuclear symmetry axis. The $\frac{\hbar}{\sqrt{mk}}$ factor accounts for the difference in the normalization of the $|(\mathbf{k}s)^{(-)}\rangle\rangle$ and $|(\alpha^{(-)}\rangle\rangle$ states:

$$\langle\langle(\mathbf{k}'s')^{(-)}|(\mathbf{k}s)^{(-)}\rangle\rangle = \delta(\mathbf{k}'-\mathbf{k})\,\delta_{s's} \tag{4}$$

$$\langle\langle\alpha'^{(-)}|\alpha^{(-)}\rangle\rangle = \delta_{\alpha'\alpha} \equiv \delta(\varepsilon'-\varepsilon)\delta(l'-l)\delta(j'-j)\delta(m'-m). \tag{5}$$

Substituting expansion (3) into (1) and summing over s yields

$$\begin{aligned}\frac{d\sigma_{\text{intr}}(E_\gamma,\vartheta')}{d\Omega'} &= \frac{\pi^2}{6}\frac{E_\gamma e^2_{\text{eff}}}{c\hbar} \sum_{\mu=\pm1}\sum_{\nu}\sum_{\nu'}\sum_{ljm}\sum_{l'j'}\sum_{l''} D^1_{\mu\nu}(\omega) D^{1*}_{\mu\nu'}(\omega)\,\times \\ &\times P_{l''}(\cos\vartheta')(-1)^{j+j'+\frac{1}{2}-m+l''}\,\hat{j}\,\hat{j}'\,\hat{l}\,\hat{l}'\,\hat{l}''^{2} \begin{pmatrix} l & l' & l'' \\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} j & l & j' \\ -m & 0 & m\end{pmatrix}\times \\ &\times \begin{Bmatrix} j & l'' & j' \\ l' & \frac{1}{2} & l\end{Bmatrix} \langle\langle\alpha^{(-)}|r'Y_{1\nu}(\theta',\varphi')|\beta\rangle\langle\langle\alpha'^{(-)}|r'Y_{1\nu'}(\theta',\varphi')|\beta\rangle^{*}\Big|_{\varepsilon'=\varepsilon},\end{aligned} \tag{6}$$

where the notation $\hat{J} = \sqrt{2J+1}$ is used (???).

Formula (6) describes angular distribution of photonucleons in the intrinsic coordinate frame and its fixed position relative to the laboratory frame is determined by the Euler angles ω. In order to obtain angular distribution of photonucleons (under adiabatic approximation) in laboratory frame one has to

1. convert the components of the spherical tensor $P_{l''}(\cos\vartheta') = \sqrt{\frac{4\pi}{2l''+1}} Y_{l''0}(\vartheta',\phi')$ that is contained in (6) to the laboratory frame:

$$Y_{l''0}(\vartheta',\phi') = \sum_{m''} D^{l''*}_{m''0}(\omega) Y_{l''m''}(\vartheta,\phi); \tag{7}$$

2. perform averaging of the obtained expression over all possible directions of the nucleus in laboratory frame, which effectively reduces to computation of an integral $\frac{1}{8\pi^2}\int\limits_0^{\pi}\sin\theta_1 d\theta_1\int\limits_0^{2\pi}d\theta_2\int\limits_0^{2\pi}d\theta_3 D^1_{\mu\nu}(\omega)D^{1*}_{\mu\nu'}(\omega)D^{l''*}_{m''0}(\omega)$;

3. after which summation over the μ, ν', and m'' quantum numbers has to be performed.

As a result the following expression for differential cross section of direct photoeffect in laboratory frame is obtained:

$$\frac{d\sigma(E_\gamma,\vartheta)}{d\Omega}=\frac{\pi^2}{6}\frac{E_\gamma e^2_{\text{eff}}}{c\hbar}\left(A_0+A_2P_2(\cos\vartheta)\right), \tag{8}$$

where

$$A_0=\frac{2}{3}\sum_{lj}\sum_{\nu m}\left|\langle\!\langle\alpha^{(-)}|r'Y_{1\nu}(\theta',\varphi')|\beta\rangle\right|^2, \tag{9}$$

$$A_2=-\sqrt{\frac{10}{3}}\sum_{lj}\sum_{l'j'}\sum_{\nu m}(-1)^{j+j'+\frac{1}{2}-m+\nu}\,\hat{j}\,\hat{j}'\,\hat{l}\,\hat{l}'\begin{pmatrix} l & l' & 2\\ 0 & 0 & 0\end{pmatrix}\times$$
$$\times\begin{pmatrix} j & 2 & j'\\ -m & 0 & m\end{pmatrix}\begin{Bmatrix} j & 2 & j'\\ l' & \frac{1}{2} & l\end{Bmatrix}\begin{pmatrix} 1 & 1 & 2\\ -\nu & \nu & 0\end{pmatrix}\langle\!\langle\alpha^{(-)}|r'Y_{1\nu}(\theta',\varphi')|\beta\rangle\times$$
$$\times\langle\!\langle\alpha'^{(-)}|r'Y_{1\nu}(\theta',\varphi')|\beta\rangle^*\Big|_{\varepsilon'=\varepsilon,m'=m}. \tag{10}$$

As expected the angular distribution of direct photonucleons associated with $E1$ photon absorption is symmetric with respect to the $\vartheta=90°$ angle between the directions of the outgoing nucleon and the incident photon.

Integration of Eq. (8) over the polar ϑ and asimuthal ϕ angles of the outgoing nucleon yields total cross section of direct photoeffect in the considered approximation:

$$\sigma(E_\gamma)=\frac{4\pi^3}{9}\frac{E_\gamma e^2_{\text{eff}}}{c\hbar}\sum_{lj}\sum_{\nu m}\left|\langle\!\langle\alpha^{(-)}|r'Y_{1\nu}(\theta',\varphi')|\beta\rangle\right|^2. \tag{11}$$

At the end of this section it should be noted that all matrix elements in (9)–(11) are calculated in the intrinsic coordinate frame x', y', z'.

II. SYSTEM OF EQUATIONS DESCRIBING COUPLED lj REACTION CHANNELS

As it was mentioned above an axial symmetric deformed optical potential $V(r,\theta)$ will be used for description of the mean field where the motion of the outgoing nucleon takes place (in subsequent discussion the prime marks of spatial variables in intrinsic coordinate frame will be omitted). It can be split into two parts:

$$V(r,\theta) = V_{\rm sp}(r) + V_{\rm def}(r,\theta), \tag{12}$$

where $V_{\rm sp}(r)$ is a usual spheric optical potential (see [2]) and $V_{\rm def}(r,\theta)$ is a polar angle θ-dependent part of the optical potential, which leads to coupling between the reaction channels for scattered states with different moments l, j.

As shown in [3] the scattered state $|(\mathbf{k}s)^{(-)}\rangle\!\rangle$ in potential (12) can be expressed in the form

$$|(\mathbf{k}s)^{(-)}\rangle\!\rangle = |(\mathbf{k}s)^{(-)}\rangle + \frac{1}{\varepsilon - H - i\rho} V_{\rm def} |(\mathbf{k}s)^{(-)}\rangle, \quad \rho \to +0, \tag{13}$$

where $H = H_{\rm sp} + V_{\rm def}$ is the total single-particle hamiltonian of the scattering problem, $H_{\rm sp} = \frac{-\hbar^2}{2m}\Delta + V_{\rm sp}$ is its spherical component, $|(\mathbf{k}s)^{(-)}\rangle$ is the scattered state with waves converging at infinity for the $H_{\rm sp}$ hamiltonian.

An important property of (13) is fulfillment of proper boundary conditions for the scattered state $|(\mathbf{k}s)^{(-)}\rangle\!\rangle$. Expansion (3) can be used to transform it into equation for partial waves $|\alpha^{(-)}\rangle\!\rangle$:

$$|\alpha^{(-)}\rangle\!\rangle = |\alpha^{(-)}\rangle + \frac{1}{\varepsilon - H - i\rho} V_{\rm def} |\alpha^{(-)}\rangle. \tag{14}$$

Here, $|\alpha^{(-)}\rangle \equiv |(\varepsilon ljm)^{(-)}\rangle$ is the scattered state of a nucleon with waves converging at infinity in the spherical mean field $V_{\rm sp}$, that is characterized by the nucleon energy ε, orbital and total angular momenta l and j, and the projection m of the angular moment on the nuclear symmetry axis. It is related to the regular solution $|\alpha\rangle \equiv |\varepsilon lmm\rangle$ of the stationary Shroedinger equation $H_{\rm sp}|\alpha\rangle = \varepsilon|\alpha\rangle$ through the following relationship

$$|\alpha^{(-)}\rangle = e^{-i\delta_\alpha}|\alpha\rangle, \tag{15}$$

where $\delta_\alpha \equiv \delta_{lj}$ is the scattered nucleon phase in the $V_{\rm sp}$ field.

Matrix elements $\langle\!\langle \alpha^{(-)}| r Y_{1\nu}(\theta,\varphi)|\beta\rangle$ that appear in the direct photoeffect cross section $\frac{d\sigma}{d\Omega}(E_\gamma, \vartheta)$ and $\sigma(E_\gamma)$ (equations (8)–(11)) describe probability amplitudes of a $E1$ transition of a nucleon from the state $|\beta\rangle$ to the scattered state $|\alpha^{(-)}\rangle\!\rangle$. It is assumed that conjugated states (Dirac's co-vectors) $\langle\!\langle \alpha^{(-)}|$ in continous spectrum and $\langle n|$ in discrete spectrum have to be orthogonal to eigenstates $|\alpha^{(-)}\rangle\!\rangle$ and $|n\rangle$ of a non-hermitian hamiltonian H, that contains the complex optical potential.

In order to construct such states we note that the states $\langle\Psi_1|, \langle\Psi_2|, \ldots$, conjugated to the eigenvectors $|\Psi_1\rangle, |\Psi_2\rangle, \ldots$ of a non-hermitian hamiltonian $\tilde{H} \neq \tilde{H}^+ = \tilde{H}^*$ meet the necessary requirements if they are defined throught the following equations:

$$\langle\Psi_i|\tilde{H} = E_i\langle\Psi_i|, \quad i = 1, 2, \ldots \tag{16}$$

Indeed, in this case $\langle\Psi_i|\tilde{H}|\Psi_k\rangle = E_i\langle\Psi_i|\Psi_k\rangle = E_k\langle\Psi_i|\Psi_k\rangle$. It follows from this fact that $\langle\Psi_i|\Psi_k\rangle = 0$ when $E_i \neq E_k$.

Equation (16) can be rewritten as (with ξ being the spin variable)

$$\sum_{\xi=\pm\frac{1}{2}} \int \langle \mathbf{r}'\xi'|\tilde{H}^*|\mathbf{r}\xi\rangle\langle\Psi_i|\mathbf{r}\xi\rangle^* d^3r = E_i^*\langle\Psi_i|\mathbf{r}'\xi'\rangle^*,$$

from which it follows that when $\tilde{H}^* \neq \tilde{H}$ the wave function $\langle\Psi_i|\mathbf{r}\xi\rangle$ of the conjugated state $\langle\Psi_i|$ is complex conjugate to a wave function of the $\tilde{H}^*$ eigenstate, and not one of the hamiltonian $\tilde{H}$.

For the regular solution $|\alpha\rangle$ of the $H_{\rm sp}$ hamiltonian this yields

$$\langle\alpha|\mathbf{r}\xi\rangle = \langle\alpha|r\rangle\langle\alpha|\hat{\mathbf{r}}\xi\rangle \equiv \langle\varepsilon lj|r\rangle\langle ljm|\hat{\mathbf{r}}\xi\rangle, \tag{17}$$

where

$$\langle\alpha|r\rangle = \langle r|\alpha\rangle \overset{r\to\infty}{\longrightarrow} \frac{i}{2r}\sqrt{\frac{2m}{\pi\hbar^2 k}}\left\{e^{-i(kr-\frac{l\pi}{2}+\delta_{lj})} - e^{i(kr-\frac{l\pi}{2}+\delta_{lj})}\right\}, \tag{18}$$

$$\langle\alpha|\hat{\mathbf{r}}\xi\rangle = (lm-\xi\,\frac{1}{2}\xi|jm)Y^*_{lm}(\theta,\varphi). \tag{19}$$

It can be easily seen that with the presented definition of conjugate states $\langle\alpha|$ the normalization condition is fulfilled:

$$\langle\alpha'|\alpha\rangle = \left(\int_0^\infty \langle\alpha'|r\rangle\langle r|\alpha\rangle r^2 dr\right)\left(\sum_\xi (l'm'-\xi\,\frac{1}{2}\xi|j'm')(lm-\xi\,\frac{1}{2}\xi|jm)\times\right.$$
$$\left.\times\int_0^{2\pi} d\varphi \int_0^\pi \sin\theta\, d\theta\, Y^*_{l'm'}(\theta,\varphi)Y_{lm}(\theta,\varphi)\right) = \delta(\varepsilon'-\varepsilon)\,\delta_{l'\,l}\,\delta_{j'j}\,\delta_{m'\,m} \equiv \delta_{\alpha'\alpha}. \tag{20}$$

The conjugate scattered state $\langle\alpha^{(-)}|$ is determined from the relationship

$$\langle\alpha^{(-)}| = e^{i\delta_\alpha}\langle\alpha|. \tag{21}$$

Now we obtain an equation for the conjugate scattering state $\langle\!\langle\alpha^{(-)}$. From (16) we obtain taking into account decomposition of the hamiltonian H in two parts

$$\langle\alpha^{(-)}|(H-\varepsilon) = \langle\alpha^{(-)}|V_{\rm def}.$$

It follows that the conjugate state $\langle\!\langle\alpha^{(-)}|$ that satisfies equation $\langle\!\langle\alpha^{(-)}|(H-\varepsilon) = 0$ can be expressed in the form

$$\langle\!\langle\alpha^{(-)}| = \langle\alpha^{(-)}| + \langle\alpha^{(-)}|\,V_{\rm def}\,\frac{1}{\varepsilon - H + i\rho}, \tag{22}$$

where the sign in front of an infinitesimal constant ρ that pushes the singular point away from the real-valued energy axis is chosen so that to ensure orthonormalization of the scattering states, as it is demonstrated below. From (14) and (22) we have

$$\langle\langle\alpha'^{(-)}|\alpha^{(-)}\rangle\rangle = \delta_{\alpha'\alpha} + \frac{\langle\alpha'^{(-)}|V_{\rm def}|\alpha^{(-)}\rangle\rangle}{\varepsilon' - \varepsilon + i\rho} + \langle\alpha'^{(-)}|\,\frac{1}{\varepsilon - H - i\rho} V_{\rm def}\,|\alpha^{(-)}\rangle.$$

Then, using the equality

$$\langle\alpha'^{(-)}|\,\frac{1}{\varepsilon - H - i\rho} = \frac{1}{\varepsilon - \varepsilon' - i\rho}\left\{\langle\alpha'^{(-)}| + \langle\alpha'^{(-)}|V_{\rm def}\frac{1}{\varepsilon - H - i\rho}\right\}$$

and once more (14) we obtain

$$\langle\langle\alpha'^{(-)}|\alpha^{(-)}\rangle\rangle = \delta_{\alpha'\alpha} + \frac{1}{\varepsilon - \varepsilon' - i\rho}\Big\{ - \langle\alpha'^{(-)}|V_{\rm def}|\alpha^{(-)}\rangle\rangle +$$
$$+ \langle\alpha'^{(-)}|V_{\rm def}|\alpha^{(-)}\rangle + \langle\alpha'^{(-)}|V_{\rm def}|\alpha^{(-)}\rangle\rangle - \langle\alpha'^{(-)}|V_{\rm def}|\alpha^{(-)}\rangle\Big\} = \delta_{\alpha'\alpha}$$

Which proves the statement.

The wave function of the scattered state $\langle\langle\alpha^{(-)}|$ has an overlap with the bound state wave function $|\beta\rangle$ only in the internal reaction region (when $r \lesssim R \approx 1.6A^{1/3}$ fm). In this limited space region spherical harmonic oscillator states $|Nljm\rangle$ can be used as basis states (the N quantum number corresponds to total number of oscillator phonons). The $\langle\langle\alpha^{(-)}|rY_{1\nu}(\theta,\varphi)|\beta\rangle$ matrix element can, therefore, be expressed as

$$\langle\langle\alpha^{(-)}|rY_{1\nu}(\theta,\varphi)|\beta\rangle = \sum_{N'l'j'} \langle\langle\alpha^{(-)}|N'l'j'm\rangle\langle N'l'j'm|rY_{1\nu}(\theta,\varphi)|\beta\rangle, \tag{23}$$

where it is taken into account that in an axially symmetric field the scattered state is described with a definite magnetic quantum number m.

The energy of the spherical oscillator quantum $\hbar\omega$ is reasonable to be chosen so as to reproduce the experimental mean-squared radius of the nucleon distribution inside the nucleus. This yields the value $\hbar\omega = 41A^{-1/3}$ MeV.

The oscillator states $|Nljm\rangle$ can be used also to aproximate the initial state $|\beta\rangle$ from which the nucleon is knocked out. Thus, within the Nilsson model [4, 5] based on the deformed oscillator potential it will be of the form

$$|\beta\rangle = \sum_{lj\in\beta} c_{lj}\,|N_\beta\, ljm_\beta\rangle, \tag{24}$$

where c_{lj} are the coefficients of expansion of the state $|\beta\rangle$ into spherical oscillator states calculated by this model.

By substituting (24) into the matrix element $\langle N'l'j'm|rY_{1\nu}(\theta,\varphi)|\beta\rangle$, we obtain

$$\langle N'l'j'm|rY_{1\nu}(\theta,\varphi)|\beta\rangle = \sum_{lj\in\beta} c_{lj}\langle N'l'|r|N_\beta l\rangle\langle l'j'm|Y_{1\nu}(\theta,\varphi)|ljm_\beta\rangle, \tag{25}$$

where the radial matrix element $\langle N'l'|r|N_\beta l\rangle$ can be calculated using the analytical expression given in [5], and the angular matrix element $\langle l'j'm|Y_{1\nu}(\theta,\varphi)|ljm_\beta\rangle$ is determined by the expression

$$\langle l_2j_2m_2|Y_{l\nu}(\theta,\varphi)|l_1j_1m_1\rangle = (-1)^{m_2-\frac{1}{2}}\sqrt{\frac{1}{4\pi}}\,\hat{l}_2\,\hat{l}_1\,\hat{j}_2\,\hat{j}_1\,\hat{l}\begin{pmatrix} l_2 & l & l_1 \\ 0 & 0 & 0 \end{pmatrix}\times$$
$$\times\begin{pmatrix} j_2 & l & j_1 \\ m_2 & -\nu & -m1 \end{pmatrix}\begin{Bmatrix} j_2 & l & j_1 \\ l_1 & \frac{1}{2} & l_2 \end{Bmatrix}. \tag{26}$$

The problem of calculation of the probability amplitudes (23) is therefore reduced to calculation of the $Nljm$ components of the conjugate scattered states $\langle\!\langle\alpha^{(-)}|$ in the internal region of the reaction.

In order to obtain the system of equations connecting this components and, thus, different lj reaction channels we reformulate expression (22) for the conjugate scattered state in a different way. Namely, using the equality

$$\frac{1}{\varepsilon - H + i\rho} = \frac{1}{\varepsilon - H_{\rm sph} + i\rho} + \frac{1}{\varepsilon - H + i\rho}V_{\rm def}\frac{1}{\varepsilon - H_{\rm sph} + i\rho},$$

we transform it to the form

$$\langle\!\langle\alpha^{(-)}| = \langle\alpha^{(-)}| + \langle\!\langle\alpha^{(-)}|V_{\rm def}\frac{1}{\varepsilon - H_{\rm sph} + i\rho}. \tag{27}$$

Let us expand the set of states $|\alpha\rangle = |\varepsilon ljm\rangle$ by including not only regular continuous spectrum solutions of the Shroedinger equation $H_{\rm sph}|\alpha\rangle = \varepsilon|\alpha\rangle$ but also eigenstates of $H_{\rm sph}$ from discrete spectrum. Then, after multiplication of Eq. (27) on the left with the oscillator state $|N'l'j'm\rangle$ and using the fullness property of the extended basis $|\alpha\rangle$ and for oscillator states $|Nljm\rangle$ (in a bounded space region where $V_{\rm def}\neq 0$ we obtain

$$\langle\!\langle\alpha^{(-)}|N'l'j'm\rangle = \delta_{ll'}\delta_{jj'}e^{i\delta_{lj}}\langle\varepsilon lj|N'l\rangle + \sum_{N''l''j''}\sum_{N_1'l_1'j_1'}\sum_{\tilde\alpha}\langle\!\langle\alpha^{(-)}|N''l''j''m\rangle\times$$
$$\times\langle N''l''j''m|V_{\rm def}|N_1'l_1'j_1'm\rangle\langle N_1'l_1'j_1'm|\frac{1}{\varepsilon - H_{\rm sph} + i\rho}|\tilde\alpha\rangle\langle\tilde\alpha|N'l'j'm\rangle, \tag{28}$$

where $\langle\varepsilon lj|N'l\rangle = \int_0^\infty\langle\varepsilon lj|r\rangle\langle r|N'l\rangle r^2\,dr$ is the radial part of the inner product $\langle\alpha|N'ljm\rangle$ and integration over the extended basis is assumed, that is, $\sum_{\tilde\alpha}\equiv\sum_{\tilde l\,\tilde j\,\tilde m}\{\int_{\tilde\varepsilon\geq 0}d\tilde\varepsilon + \sum_{Re\tilde\varepsilon+<0}\}$.

The sum can be expressed as

$$\sum_{\tilde\alpha}\langle N_1'l_1'j_1'm|\frac{1}{\varepsilon - H_{\rm sph}+i\rho}|\tilde\alpha\rangle\langle\tilde\alpha|N'l'j'm\rangle =$$

$$= \delta_{l'l_1'}\delta_{j'j_1'}\left\{\int\limits_0^\infty \frac{\langle N_1'l'|\,\tilde\varepsilon l'j'\rangle = \langle\tilde\varepsilon l'j'|N'l'\rangle\, d\tilde\varepsilon}{\varepsilon-\tilde\varepsilon+i\rho} + \sum_{Re\tilde\varepsilon<0}\frac{\langle N_1'l'|\tilde\varepsilon l'j'\rangle\langle\tilde\varepsilon\, l'j'|N'l'\rangle}{\varepsilon-\tilde\varepsilon}\right\}. \qquad (29)$$

The second term in this expression can be neglected due to its small value in the region where the "direct photoeffect resonance" is formed due to a large value of $|\varepsilon-\tilde\varepsilon|$ for discrete states. The first term inside curly braces can be expressed as

$$f_{N_1'N'l'j'}(\varepsilon) = P\int\limits_0^\infty \frac{N_1'l'|\varepsilon'l'j'\rangle\varepsilon'l'j'|N'l'\rangle\, d\varepsilon'}{\varepsilon-\varepsilon'} - i\pi N_1'l'|\varepsilon l'j'\rangle\varepsilon l'j'|N'l'\rangle. \qquad (30)$$

It follows from (29) and (30) that relationship (28) can be rewritten as system of algebraic equations in $Nljm$ components of the scattered state $\langle\langle\alpha^{(-)}|$ in the internal region of the reaction:

$$\sum_{N''l''j''} W_{N'l'j',\,N''l''j''}(\varepsilon,m)\langle\langle\alpha^{(-)}|N''l''j''m\rangle = -\delta_{ll'}\delta_{jj'}e^{i\delta_{lj}}\langle\varepsilon lj|N'l\rangle, \qquad (31)$$

where the W matrix elements are defined by the expression

$$W_{N'l'j',\,N''l''j''}(\varepsilon,m) = \sum_{N_1'} f_{N_1'N'l'j'}(\varepsilon)''l''j''m|V_{\rm def}|N'l'j'm\rangle -$$

$$-\ \delta_{N'N''}\delta_{l'l''}\delta_{j'j''}. \qquad (32)$$

The number of significant components $\langle\langle\alpha^{(-)}|Nljm\rangle$ of the scattered state in the internal reaction region, which determines the effective dimension of the W matrix at fixed values of energy $\varepsilon = E_\gamma - B_{\rm thr}$ and angular moment projection $m = \nu + m_\beta$ (see (11)), is not very large due to:

1. conservation of parity: $(-1)^l = (-1)^N = -(-1)^{N_\beta}$;

2. finiteness of the orbital moments of the nucleon knocked out the peripheral region of a nucleus: $0 \le l \lesssim l_{\rm max} = kR$;

3. satisfied conditions: $l \le N \le N_{\rm max}$, $|l-\frac{1}{2}| \le j \le l+\frac{1}{2}$, $j \ge |m|$ (where one can choose $N_{\rm max} \approx l_{\rm max}+4$, which allows the $\langle\langle\alpha^{(-)}|Nljm\rangle$ components with large orbital moments to be correctly described).

The above limitations result in an optimal number of dimensions of the system (32) not exceeding 100 when $E_\gamma \lesssim 50$ MeV.

III. DEFORMED OPTICAL POTENTIAL

The spherical optical potential takes the form [2]:

$$V_{\rm sph}(r) = -(V_1 + iW_1)\,\frac{1}{1+\exp[(r-R_1)/a_1]} - \\ -4iW_2\,\frac{\exp[(r-R_2)/a_2]}{\{1+\exp[(r-R_2)/a_2]\}^2} - (V_3+iW_3)\left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{2a_3 r}\times \\ \times\,\frac{\exp[(r-R_3)/a_3]}{\{1+\exp[(r-R_3)/a_3]\}^2}\,\mathbf{s}\!\cdot\!\mathbf{l} + V_{\rm Coul}(r), \tag{33}$$

where the first two terms describe the nuclear interaction, the third term corresponds to the spin-orbit interaction, and the fourth term

$$V_{\rm Coul}(r) = \begin{cases} \dfrac{3}{2}\dfrac{qZe^2}{R_{\rm Coul}}\left(1-\dfrac{r^2}{3R_{\rm Coul}^2}\right) & \text{if } r \le R_{\rm Coul}, \\ \dfrac{qZe^2}{r} & \text{if } r \ge R_{\rm Coul} \end{cases} \tag{34}$$

corresponds to the Coulomb interaction (q is 0 for a neutron, and 1 for a proton, $R_{\rm Coul} = r_{\rm Coul}A^{1/3}$ is the Coulomb radius).

If a nucleus is axially symmetric ellipsoidal-shaped with semi-major and semi-minor axes c and d directed, respectively, along to the nuclear symmetry axis and orthogonal to it, then its surface can be described with a function

$$R(\theta) = R_0(1-\eta)^{1/6}(1-\eta\cos^2\theta)^{-1/2}, \tag{35}$$

where R_0 is the non-deformed radius ($R_0^3 = cd^2$), $\eta = (c^2-d^2)/c^2$ is a parameter that characterizes the deformation connected with nuclear quadrupole deformation parameter $\delta = \frac{3}{2}(c^2-d^2)/(c^2+2d^2)$ with the relationship

$$\eta = \frac{2\delta_0}{1+4\delta_0/3}\,.$$

The radial and angular dependencies of the nuclear component in the mean field is in a tight correlation with the density distribution of nuclear matter. If the thickness of the diffused surface layer of the nucleus is small in comparison with its radius, then variation of this mean field component due to deformation can be taken into account by introduction of angular dependence of R_1 and R_2 in (33) analogously to (35), substituting R_0 with R_1 and R_2, respectively.

As in [6] we neglect the effect of deformation on spin-oribit interaction and for the Coulomb field that strongly affects proton scattering we will use in the deformed optical potential $V(r,\theta)$

the expression obtained in [7]:

$$V_{\text{Coul}}(r,\theta)=\begin{cases}\dfrac{3}{2}\dfrac{qZe^2}{R_{\text{Coul}}}\left[\left(1-\dfrac{r^2}{3R_{\text{Coul}}^2}\right)+\right.\\ \left.+\displaystyle\sum_{n=1}^{\infty}\left(\alpha_n+\beta_n\frac{r^2}{R_{\text{Coul}}^2}P_2(\cos\theta)\right)\eta^n\right] & \text{if } r\le R(\theta),\\ qZe^2\left[\dfrac{1}{r}+\displaystyle\sum_{n=1}^{\infty}\sum_{l=0}^{n}\gamma_{nl}\frac{R_{\text{Coul}}^{2l}}{r^{2l+1}}P_{2l}(\cos\theta)\eta^n\right] & \text{if } r>R(\theta),\end{cases} \tag{36}$$

where $R(\theta)$ is determined from (35) by means of a substitution $R_0 \to R_{\text{Coul}}$ and the coefficients $\alpha_n, \beta_n, \gamma_{nl}$ are defined by the expressions

$$\alpha_n=\sum_{k=0}^{n}\frac{(-1)^k\Gamma_k(1/3)}{(2n-2k+1)k!}\,,\qquad \beta_n=\frac{2}{(2n+1)(2n+3)}\,,$$
$$\gamma_{nl}=\frac{3}{(2l+3)!!}\sum_{k=l}^{n}\frac{(-1)^{n-k}\Gamma_{n-k}\left(\frac{2l+3}{6}\right)(2l+2k+1)!!}{2^{2l+k}(n-k)!\,k!}\times$$
$$\times\sum_{m=0}^{l}\frac{(-1)^m(4l-2m)!}{m!\,(2l-m)!\,(2l-2m)!\,(2l-2m+2k+1)}\,. \tag{37}$$

(Here $\Gamma_j(x)=x(x-1)\ldots(x-j+1)$ with $j=1,2,\ldots$; $\Gamma_0(x)=1$.)

The series in (36) converge when $|\eta|<1$, thus describing the Coulomb component of the optical potential $V(r,\theta)$ at quadrupole deformations $-0.3<\delta<1.5$. When $\delta \lesssim 0.4$ only the first ten elements of the series need to be considered.

According to definition (12) the potential $V_{\text{def}}(r,\theta)$ that was previously used in derivation of the system of equation for the coupled lj-channels is given by the expression

$$V_{\text{def}}(r,\theta)=V(r,\theta)-V_{\text{sph}}(r). \tag{38}$$

It can be expanded into spherical harmonics

$$V_{\text{def}}(r,\theta)=\sum_{\lambda}v_\lambda(r)Y_{\lambda0}(\theta), \tag{39}$$

where λ takes the values $0,2,4,\ldots$ and the $v_\lambda(r)$ function is defined by the expression

$$v_\lambda(r)=4\pi\int_0^1 V_{\text{def}}(r,\theta)Y_{\lambda0}(\theta)\,d(\cos\theta). \tag{40}$$

Using expansion (39) one can rewrite the matrix element $\langle N''l''j''m|V_{\text{def}}|N'l'j'm\rangle$ from (32) in the form

$$\langle N''l''j''m|V_{\text{def}}|N'l'j'm\rangle=\sum_{\lambda}\lambda N''l''|v_\lambda(r)|N'l'\rangle\langle l''j''m|Y_{\lambda0}(\theta)|l'j'm\rangle, \tag{41}$$

where the radial matrix element $\langle N''l''|v_\lambda(r)|N'l'\rangle$ is calculated numerically and the angular matrix element $\langle l''j''m|Y_{\lambda0}(\theta)|l'j'm\rangle$ according to (26).

IV. APPLICATION TO DESCRIPTION OF DIRECT (γ,P) REACTIONS ON ^{160}GD, ^{184,186}W

The above described model was used to calculate cross sections of direct photonuclear reactions (γ,p) in the case of ^{160}Gd, ^{184,186}W where in the literature there are available experimental data [8, 9] obtained in a bremsstrahlung beam using activation technique. The calculation was performed in the energy range $E_\gamma = 0..60$ MeV with energy pitch size $h = 0.1$ MeV. The obtained cross sections $\sigma(E_i), i = 1, 2, \dots$ were then averaged over the energy window $\Delta = 2$ MeV:

$$\bar{\sigma}(E) = \sum_i \frac{1}{2\pi} \frac{\Delta h}{(E_i - E)^2 + (\Delta/2)^2} \sigma(E_i), \quad (42)$$

so as to equalize the energy resolution of theoretical and experimental procedures.

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